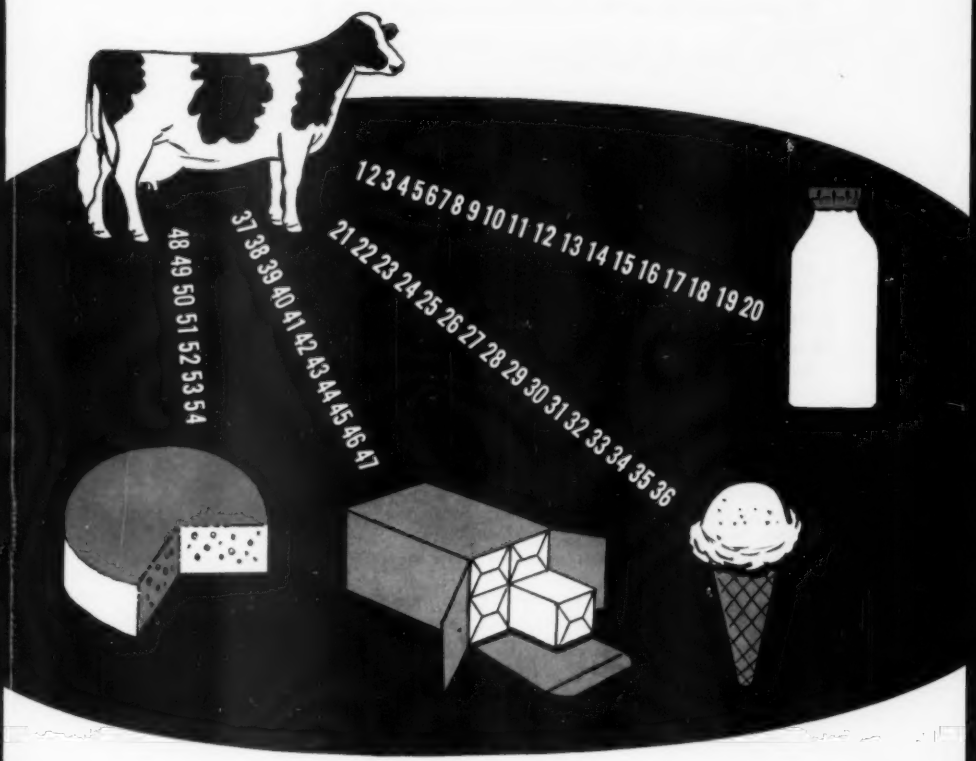
DAIRY ARITHMETIC

BY W. K. JORDAN AND R. P. MARCH



CORNELL EXTENSION BULLETIN 675

NEW YORK STATE COLLEGE OF AGRICULTURE

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DAIRY ARITHMETIC

W. K. JORDAN AND R. P. MARCH

The principles of mathematics find wide application in the problems that arise in the routine processing and handling of milk and milk products.

This bulletin shows, by specific examples, how many of these computations are made. The examples are grouped under the following headings: Fluid Milk and Cream, Condensed Milk, Butter, Miscellaneous Problems, Cheese, and Ice Cream.

FLUID MILK AND CREAM

Converting Volume to Weight and Weight to Volume

In converting milk weight to volume or volume to weight it is helpful to know that the weight of a quart of average milk is 2.15 pounds.

Values for other products, as given in the August 1, 1955, edition of "Rules and Regulations, Market Administrator, New York Metropolitan Milk Marketing Area" are as follows:

Product	Percent fat	Pounds per quart	Weight per 40-qt. can
Skim milk	—	2.15	—
Milk	—	2.15	85.00
Cream	18	—	85.26
Cream	25	—	84.70
Cream	36	—	83.94
Cream	40	—	83.74
Cream	50	—	83.10

These factors are required to be used in those instances where the records of handlers subject to the marketing orders do not reveal specific butterfat tests or specific weights. These figures have been established on the basis of data, views, and arguments presented at public meetings.

Values given by the National Bureau of Standards and published in the *Journal of Agricultural Research*, Vol. III, No. 3 (U.S. Department of Agriculture), Table II, p. 257, are as follows:

Product	Percent fat	Pounds per gallon at 50°F.	Pounds per gallon at 68°F.
Skim milk	0.025	8.65	8.63
Milk	3.00	8.62	8.60
Milk	4.00	8.61	8.59
Milk	5.00	8.60	8.58
Cream	18.00	8.49	8.44
Cream	25.00	8.45	8.39
Cream	36.00	8.40	8.31
Cream	40.00	8.38	8.28

Note that the weight of a given volume of milk or cream varies slightly with its temperature.

Conversion of weight to volume or volume to weight simply necessitates multiplication or division, using data from the above tables given previously.

Example 1

40 quarts of milk are equal to how many pounds?
1 quart of milk weighs 2.15 pounds.

Answer: 40 quarts weigh 2.15 x 40 = 86 pounds.

Example 2

100 pounds of milk equals how many quarts?

Answer: 2.15 pounds of milk equals 1 quart; therefore, in 100 pounds there would be 100 ÷ 2.15 = 46.5 quarts.

Example 3

5 gallons of 36 percent cream weigh how many pounds?

Answer: 1 gallon of cream equals 8.4 pounds; therefore, 5 gallons of cream weigh 8.4 x 5.0 = 42 pounds.

Percentage

The word percent means "by the hundred". The symbol % means percent. Thus, 10 percent (%) is $\frac{10}{100}$ or 0.10 when written as a decimal.

When writing a percentage in a mathematical problem *always write it as a decimal*. For example: 3% milk = 0.03; 20% cream = 0.20; 0.01% skim = 0.0001.

Computing the Weight of Various Components of Milk in Dairy Products

To find the weight of a component of a dairy product it is necessary to know the total weight of the product and the percentage of its component in question. Then the following formula can be used:

Formula 1

Weight of product x test (percent of component written as a decimal) = weight of component.

Example 4

How many pounds of fat are there in 196 pounds of 40.5 percent cream?

Answer: 196 x 0.405 = 79.38 pounds of fat

Example 5

How many pounds of solids-not-fat (SNF) are there in 96.5 pounds of skim milk containing 8.9 percent SNF?

Answer: 96.5 x 0.089 = 8.59 pounds of solids-not-fat

Example 6

The composition of milk is: water 87.1 percent, milkfat 3.9 percent, casein 2.5 percent, albumin 0.7 percent, milk sugar 5.1 percent, and ash 0.7 percent. Compute the pounds of each constituent in 955 pounds of this milk.

Answers:
955 x 0.871 = 831.81 pounds of water
955 x 0.039 = 37.25 pounds of milkfat
955 x 0.025 = 23.88 pounds of casein
955 x 0.007 = 6.69 pounds of albumin
955 x 0.051 = 48.71 pounds of milk sugar
955 x 0.007 = 6.69 pounds of ash

Example 7

The composition of a sample of butter is: milkfat 80.3 percent, water 16.9 percent, salt 2.0 percent, and casein 0.8 percent. Compute the pounds of each component in 825 pounds of this butter.

Answers:
825 x 0.803 = 662.48 pounds of milkfat
825 x 0.169 = 139.43 pounds of water
825 x 0.020 = 16.50 pounds of salt
825 x 0.008 = 6.60 pounds of casein

Example 8

The composition of an American cheddar cheese is: water 38 percent, milkfat 33 percent, milk-solids-not-fat 29 percent. Compute the pounds of each component in 34 pounds of this cheese.

Answers:
34 x 0.38 = 12.92 pounds of water
34 x 0.33 = 11.22 pounds of milkfat
34 x 0.29 = 9.86 pounds of milk-solids-not-fat

Computing the Percentage of Various Components of Milk in Dairy Products

To find the percentage of any component of a dairy product it is necessary to know the weight of the component and the total weight of the dairy product. Then the following formula can be used:

Formula II

Weight of component ÷ weight of product = percentage of component (written as a decimal).

Example 9

250 pounds of cream contained 75 pounds of milkfat. What did the cream test (what is the percentage of fat in the cream)?

Answer: 75 ÷ 250 = 0.30 or 30 percent fat

Proof: 250 x 0.30 = 75

Example 10

845 pounds of ice cream mix contain 122.53 pounds of sugar and 4.23 pounds of gelatin. What is the percentage of sugar and of gelatin in this mix?

Answer: A. 122.53 ÷ 845 = 0.145 or 14.5 percent sugar
B. 4.23 ÷ 845 = 0.005 or 0.5 percent gelatin

Example 11

705 pounds of evaporated milk contain 517.7 pounds of water, 58.0 pounds of fat, 47.3 pounds of protein, 71.4 pounds of lactose and 10.6 pounds of ash. What is the percentage of each component in the evaporated milk?

Answers: 517.7 ÷ 705 = 0.734 or 73.4 percent water
58.0 ÷ 705 = 0.082 or 8.2 percent fat
47.3 ÷ 705 = 0.067 or 6.7 percent protein
71.4 ÷ 705 = 0.101 or 10.1 percent lactose
10.6 ÷ 705 = 0.015 or 1.5 percent ash

Computing the Weight of a Dairy Product if the Weight and Percentage of Any One Component Is Known

The weight of any dairy product can be calculated if the weight and percentage of any one component is known.

Formula III

Weight of component ÷ percent of component (written as a decimal) = weight of product.

Example 12

If 190.6 pounds of sugar were used to make sweetened condensed milk containing 42.9 percent sugar, how many pounds of sweetened condensed milk were obtained?

Answer: 190.6 ÷ 0.429 = 444.3 pounds of sweetened condensed milk.

Example 13

How many pounds of milk testing 4.5 percent fat would be required to furnish 157.5 pounds of fat?

Answer: 157.5 ÷ 0.045 = 3500 pounds of 4.5 percent milk

Proof: 3500 x 0.045 = 157.5 pounds of fat

These three formulas, I, II, and III, can be used in various combinations to solve many problems.

Example 14

How many pounds of 22 percent cream can be obtained from 3500 pounds of 4 percent milk?

Solution: In 3500 pounds of 4 percent milk there are 140 pounds of fat (3500 x 0.04 = 140). Formula I is applied here. This amount of fat will be contained in the 22 percent cream. The next step is to find the number of pounds of cream testing 22 percent that can be made from 140 pounds of fat. Formula III is applied here. Therefore, 140 ÷ 0.22 = 636.36 pounds of cream.

Proof: 636.36 x 0.22 = 140 pounds of fat.

Example 15

How many pounds of milk testing 3.8 percent fat will be required to make 10 gallons of 40 percent cream?

Solution: Ten gallons of 40 percent cream weigh 83.74 pounds. Now apply formula I to determine the amount of milkfat in the can of cream. 83.74 x 0.40 = 33.50 pounds of fat. 33.50 pounds of fat ÷ 0.038 (test of milk) = 881.58 pounds.

Only One, Not Three Formulas Are Needed

Although three formulas have been given in this section, only one formula is really needed because by simple algebra the other two can quickly be written. (See Appendix page 37 for instructions in elementary algebra).

If one of the formulas is remembered it can be used to solve any problem when two of the constituents of the equation are known.

Example 16

If you have the problem, how many pounds of fat are there in 200 pounds of 4 percent milk, substitution in the formula is as follows: 200 x 0.04 = 8 pounds of fat. If the problem were, what is the total weight of 4 percent milk containing 80 pounds of fat, substitution in the formula would be as follows: ? x 0.04 = 8

Then rewriting: $? = \frac{8}{0.04} = 200$ pounds of milk

If the problem were, what is the fat test of 200 pounds of milk containing 8 pounds of fat, substitution in the formula would be as follows: 200 x ? = 8

Then rewriting: $? = \frac{8}{200} = 0.04$ or 4 percent fat

Suppose you remember the formula or equation: Weight of product x test (written as a decimal) = weight of component. Then if the product is 200 pounds of milk testing 4 percent fat, the weight of component is: 200 x 0.04 = 8 pounds of fat. If there were 72 pounds of fat in milk testing 1.8 percent fat, the total weight of milk is: weight of product x 0.018 = 72 or weight of product = 72 ÷ 0.018 = 400 pounds. If 150 pounds of milk contain 4.5 pounds of fat, the test of the milk is: 150 x test = 4.5; test = 4.5 ÷ 150 = 0.03 or 3 percent fat.

Standardizing Milk and Cream

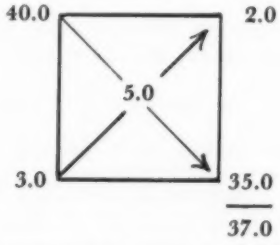
Standardizing milk or cream consists of raising or lowering the butterfat content to a desired value by the addition of milk or cream with a higher or lower percentage of butterfat than that of the material to be standardized. The solving of a problem involves finding the relative amounts of the original material and the standardizing material to mix together to give a product with the desired fat content. Once these relative amounts have been determined it is easy to calculate the exact amounts of each to mix together to give a certain weight of finished product or the exact amount of standardizing material needed to use up a given weight of milk or cream. A simple scheme, the Pearson's square, can be used to calculate the relative amounts of materials involved in a standardization problem. Remember that all measurements based on these calculations are by weight and not by volume. The scheme is as follows:

Draw a square and place in the center of it the percentage of milkfat desired. Place at the left-hand corners of the square the percentages of milkfat in the materials to be mixed. Next subtract the number in the center from the larger number at the left side of the square and place the remainder at the diagonally opposite right-hand corner. Subtract the smaller number on the left-hand side from the number in the center and place the remainder at the diagonally opposite right-hand corner. The numbers on the right-hand side now represent the number of parts of each of the original materials that must be blended to make a product with a fat test given by the number in the middle of the square. The number at the upper right corner refers to the parts of material whose fat test was placed at the upper left corner and the number at the lower right corner refers to the parts of material whose fat test was placed at the lower left corner. If the numbers on the right are added, the sum represents the parts of finished product, with fat test given by the number in the middle of the square, that will be obtained.

Example 17

How many parts by weight of 40 percent cream and of 3 percent milk must be mixed to make milk testing 5 percent?

Solution:

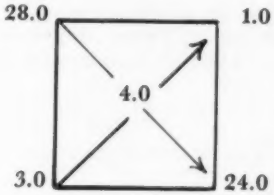


Place the desired fat content, 5 percent, in the center of the square and the fat contents of the available materials, 40 percent and 3 percent on the left side of the square. Subtract 5 from 40 and place the difference, 35, at

the corner diagonally opposite 40. Subtract 3 from 5 and place the difference, 2, at the corner diagonally opposite 3. Add 2 and 35 to get 37. The Pearson's square now tells us that when 2 parts of 40 percent cream are mixed with 35 parts of 3 percent milk, 37 parts of 5 percent milk are obtained.

Example 18

How many pounds of 28 percent cream and 3 percent milk will be required to make 500 pounds of a mixture testing 4 percent? In this problem, a definite number of pounds of the mixture is required.



Solution: Set up the square as indicated. It shows that for each 25 pounds of finished product we need 1 pound of 28 percent cream and 24 pounds of 3 percent milk. For 500 pounds of the product we will need $\frac{500}{25}$ or 20 times as much as for 25 pounds of product. The weight of cream needed is therefore: 20 x 1 = 20 pounds.

We also need 20 times as much 3 percent milk to make 500 pounds of product as was needed for 25 pounds of product, or the weight of milk needed is:

Answer: 20 x 24 = 480 pounds

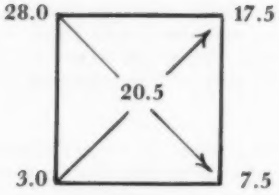
The combined weights of 20 pounds of 28 percent cream and 480 pounds of 3 percent milk gives 500 pounds of milk testing 4 percent fat.

Proof: In working problems in standardization it is always wise to prove the answer, for this is the best method of checking the work for mistakes. According to the conditions of the problem, there would be 500 pounds of 4 percent milk. This amount of milk would contain 20 pounds of milkfat (500 x 0.04 = 20). According to the results the 500 pounds would be made up of 480 pounds of 3 percent milk and 20 pounds of 28 percent cream. The 480 pounds of 3 percent milk would contain 14.4 pounds of milkfat (480 x 0.03 = 14.4). The 20 pounds of 28 percent cream would contain 5.6 pounds of milkfat (20 x 0.28 = 5.6). 14.4 + 5.6 = 20.

Since the 500 pounds contain 20 pounds of milkfat, and the materials of which the 500 pounds are made up furnish the 20 pounds of milkfat, the problem is solved correctly.

Example 19

How many pounds of 3 percent milk must be mixed with 150 pounds of 28 percent cream to make a mixture testing 20.5 percent?



Method of Calculation: Set up the square as indicated. It shows that for each 17.5 pounds of 28 percent cream, we must add 7.5 pounds of 3.0 percent milk to make cream with 20.5 percent fat. Since we have 150 pounds of the 28 percent cream, we will need $\frac{150}{17.5}$ or 8.571 times as much 3 percent milk as we did for each 17.5 pounds of the cream. The amount of 3 percent milk needed is therefore:

Answer: 8.571 x 7.5 = 64.28 pounds

Proof:

150 pounds 28 percent cream contain	150	x 0.28 =	42.0 pounds fat
64.28 pounds 3 percent milk contain	64.29	x 0.03 =	1.93 pounds fat
214.28 pounds product contain			43.93 pounds fat

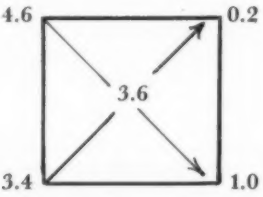
This should represent 20.5 percent fat if the work is correct.

$$\frac{43.93}{214.28} \times 100 = 20.5 \text{ percent}$$

Example 20

Mr. A has 25 Holstein cows giving a herd test of 3.4 percent milkfat. How many Jersey cows testing 4.5 percent milkfat must be added to the herd to bring the test up to 3.6 percent milkfat? Each of the 25 Holstein cows produces 40 pounds of milk per day. Each Jersey cow produces 25 pounds of milk per day.

Solution: The Holstein cows produce 25 x 40 = 1000 pounds per day. The problem now becomes one of finding the weight of 4.6 percent milk needed to standardize 1000 pounds of 3.4 percent milk to 3.6 percent fat. Set up the square as indicated:



It shows that for each 1.0 pound of 3.4 percent milk, 0.2 pound of 4.6 percent milk is needed to standardize to 3.6 percent fat. Since we have 1000 pounds of the 3.4 percent milk, we will need $\frac{1000}{1}$ or 1000 times as much of the 4.6 percent milk as we did for each pound of the 3.4 percent milk.

The amount needed is therefore: 1000 x 0.2 = 200 pounds.

Since each Jersey cow gives 25 pounds per day, we need $\frac{200}{25}$ = 8 cows.

Proof: 1000 pounds 3.4 percent milk contain
1000 x 0.034 = 34.0 pounds fat
200 pounds 4.6 percent milk contain
200 x 0.046 = 9.2 pounds fat
1200 pounds of mixed milk contain 43.2 pounds fat

This represents a fat content in the mixture of $\frac{43.2}{1200}$ x 100 = 3.6 percent

CONDENSED MILK

Standardizing Milk to Obtain a Condensed Milk of a Definite Composition

Example 21

It is desired to make plain condensed milk from 6500 pounds of whole milk testing 3.5 percent milkfat and 8.8 percent milk-solids-not-fat (msnf). The resulting condensed milk product is to contain 7.9 percent milkfat, and 18.2 percent msnf. To obtain this desired composition, it may be necessary to adjust the composition of the whole milk before condensing it. Assume that the available standardizing agents are cream containing 40 percent milkfat and 5.4 percent msnf, and skim milk containing 0.2 percent milkfat and 9 percent msnf.

Under the above conditions:

1. How much of either the cream or the skim milk should be added to the whole milk so that it will be possible to make the product with the desired composition?
2. How much finished product will be obtained?
3. How much water must be evaporated from the milk to give the desired concentration?

(1). The ratio, or proportion of msnf to milkfat must be determined. This is done by dividing the percentage of msnf (8.8) by the percentage of milkfat (3.5). Applying this calculation, the ratio of msnf to milkfat in the whole milk is 8.8 ÷ 3.5 = 2.51. This shows that for every part by weight of milkfat in this milk there are 2.51 parts of msnf.

The ratio of the msnf to the milkfat in the condensed milk may be obtained in the same way: 18.2 percent msnf ÷ 7.9 percent fat = 2.30, the msnf to fat ratio in the condensed milk. This means that, in the condensed product, for each 1 part of milkfat there should be 2.30 parts of msnf.

By comparing the msnf to fat ratios of the whole milk and the condensed milk, it will be seen that the msnf to fat ratio of the fluid milk is greater. This indicates that the amount of milkfat in the fluid milk is proportionately less than is desired in the condensed milk; and that consequently milkfat must be added to the whole milk before condensing in order to yield a condensed product of the desired composition.

The percentage of milkfat that should accompany the 8.8 percent of msnf in the whole milk in order to provide the desired balance may be obtained by dividing the percentage of msnf in the whole milk by the desired ratio in the condensed milk. 8.8 ÷ 2.30 = 3.826, the desired percentage of fat in the whole milk.

The difference between this desired percentage of fat in the whole milk and the original 3.5 percent fat content of the milk is 0.326 percent (3.826 − 3.50 = 0.326), the required increase in percentage of fat in the whole milk. Therefore, 21.19 pounds of milkfat will be required (6500 pounds of whole milk x 0.326 percent = 21.19). The simplest method of satisfying this shortage would be, of course, to add 21.19 pounds of milkfat. Two difficulties appear, however: the pure fat is not commonly available, and special means for its incorporation would be necessary. Some more common and more easily combined material should be used, and 40 percent cream is therefore chosen.

However, any addition of 40 percent cream also will introduce 5.4 percent msnf, and a portion of the fat will be required to balance these accompanying solids in the desired ratio. The percentage of fat necessary for this purpose may be determined by dividing 5.4 percent by the ratio desired. Thus: 5.4 percent msnf in the cream ÷ 2.30 (msnf-fat ratio in the condensed milk) = 2.35 percent. Of the 40 percent of fat in the cream, therefore, only 37.65 percent (40 − 2.35 = 37.65 percent) is available for standardizing.

The amount of cream necessary to provide the additional fat needed may now be computed by dividing 21.19 pounds milkfat by the standardizing value of the cream, as follows: 21.19 ÷ 37.65 percent = 56.28 *pounds of 40 percent cream required.*

(2) The weight of the finished product is readily determined upon the basis of solids constituents involved[1]. It is necessary, therefore, to compute the weights of these solids.

The 6500 pounds of milk testing 3.5 percent fat and 8.8 percent msnf will contain 227.5 pounds of milkfat (6500 x 3.5 percent = 227.50) and 572 pounds of msnf (6500 x 8.8 percent = 572). The 56.28 pounds of cream testing 40 percent fat and 5.4 percent msnf will contain 22.51 pounds of milkfat (56.28 x 40 percent = 22.51), and 3.04 pounds of msnf (56.28 x 5.4 percent = 3.04). The following summary expresses the composition of the standardized milk before it is condensed.

	Pounds of materials	Pounds of fat	Pounds of msnf
Original milk	6500.00	227.50	572.00
Added cream	56.28	22.51	3.04
Standardized milk	6556.28	250.01	575.04

[1]In computations involving whole milk the weight of the fat is commonly used, as in this illustration. The weight and percentage of the msnf, however, or the weight and percentage of the total milk solids may be used. In problems concerning skim-milk products this is often desirable.

As the 250.01 pounds of fat are to be contained in a condensed milk containing 7.9 percent of fat, the weight of the condensed milk may be found by applying formula III (page 6) as follows: 250.01 ÷ 7.9 percent = 3164.7 *pounds of finished product.*

(3) The amount of water to be removed by evaporation may now be found by determining the difference between the weight of the standardized fluid materials and the weight of the condensed milk: 6556.28 pounds of standardized milk – 3164.7 pounds of finished product = 3391.58 *pounds of water to be evaporated.*

Proof: It has already been shown in part (1) of this solution that the desired ratio of msnf to milkfat in the finished product is 2.3. Therefore, the milk that is to be condensed must contain this constituent ratio after standardization. Any subsequent change resulting from the addition or removal of water will not alter this ratio, but will merely vary the degree of concentration. The accuracy of the standardization, therefore, may be checked by again computing the msnf ratio, using the totals shown in the summary in part (2) of the solution: 575.04 pounds of msnf ÷ 250.01 pounds of fat = 2.3. This calculation shows an existing ratio of 2.3 parts of msnf to each 1 part of fat, and proves the solution of the problem to be correct.

Example 22

From 6500 pounds of whole milk containing 4 percent fat and 8.6 percent msnf, make sweetened-condensed milk containing 8.5 percent fat, 19.6 percent msnf, and 42.5 percent cane sugar. The available adjusting agents for any necessary standardizing of the fluid milk are cream containing 40 percent fat and 5.4 percent msnf, and skim milk containing 0.2 percent fat, 9 percent msnf, and cane sugar.

To conform with the above standards:

1. Is standardization of the milk necessary, and if so, how much of what available material must be added?
2. How much sugar must be added to the milk to provide the desired 42.5 percent in the finished product?
3. What should be the weight of the finished product?
4. How much water must be evaporated during the condensing operation?

(1) First, the ratios, or proportions, of msnf to fat in both the available whole milk and the desired condensed milk must be determined. Dividing the percentage of msnf in the whole milk by the percentage of milkfat, the ratio of msnf to milkfat is found to be 2.15. (8.6 percent msnf ÷ 4 percent fat = 2.15, the msnf-fat ratio in whole milk).

Similarly, dividing the percentage of msnf in the condensed milk by the percentage of accompanying milkfat shows the desired ratio to be 2.31 (19.6 percent msnf ÷ 8.5 percent = 2.31, the msnf-fat ratio in the condensed milk).

These figures show that the whole milk contains 2.15 parts by weight of msnf for each 1 part of milkfat, while the ratio desired in the condensed product is 2.31 parts of msnf for each 1 part of milkfat. A comparison of these ratios shows that the whole milk, having the lower ratio, is deficient in msnf. This fault should be corrected prior to the condensing operation, and for the purpose *fluid skim milk is to be added.*

The percentage of msnf that should accompany the 4 percent fat in the whole milk to provide the desired balance, may be found by multiplying the percentage of fat in the whole milk by the desired ratio: 4 x 2.31 = 9.24 percent msnf required in the whole milk.

Subtracting the percentage of msnf (8.6) in the original milk from the required percentage (9.24), and multiplying the milk weight by the figure obtained, shows that 41.6 pounds of msnf must be supplied by added skim milk: 9.24 − 8.6 = 0.64 percent msnf that must be supplied. Therefore, 6500 pounds whole milk x 0.64 percent = 41.6 pounds msnf to be supplied.

The skim milk, however, contains 0.2 percent fat, and a portion of the msnf will be required to balance this accompanying fat in the desired ratio. The percentage of msnf necessary for this purpose may be found by multiplying the fat content of the skim milk by the desired ratio: 0.2 percent x 2.31 (msnf-fat ratio in the condensed milk) = 0.46 percent msnf needed to accompany the 0.2 percent fat.

Of the 9 percent msnf in the skim milk there is, therefore, only 8.54 percent available for standardizing (9 − 0.46 = 8.54). The amount of skim milk necessary to provide the additional msnf needed may now be determined by dividing the weight of msnf needed by the standardizing value of the skim milk as follows: 41.6 pounds msnf needed ÷ 8.54 percent available msnf = 487.12 *pounds of skim milk* to be added to the whole milk.

(2) The next step is to compute the amount of sugar that must be added to the whole milk to equal 42.5 percent of the condensed product. The simplest way to do this is to base the computation upon the ratio between the sugar and the milkfat. (See footnote, Example 21, page 12).

Since the finished product is to contain 8.5 percent fat and 42.5 percent cane sugar, the ratio of sugar to fat is 5:1 (42.5 ÷ 8.5 = 5). This means that for every 1 pound of fat, 5 pounds of sugar should be added. The combined materials will contain 260.97 pounds of milkfat. (6500 pounds of whole milk x 4 percent fat = 260 pounds of milkfat in the original milk; 456.67 pounds of added skim milk x 0.20 percent fat = 0.97 pound of milkfat in the added skim milk; 260 + 0.97 = 260.97 total pounds of milkfat). The computation is completed by multiplying the total weight of milkfat by the previously determined ratio: 260.97 total pounds fat x 5 (sugar-fat ratio) = 1304.85 *pounds of sugar required.*

(3) The weight of the finished product may be computed, as in problem 21, upon the basis of the fat content of the milk. The preceding calculation (2) has shown the fat content of the combined materials to be 260.97 pounds. This weight of fat is to be contained in a condensed milk containing 8.5 percent fat. Again applying formula III (page 6): 260.97 total pounds fat ÷ 8.5 percent fat = 3070.24 *pounds of sweetened-condensed milk*, the finished product.

(4) The amount of water to be removed by evaporation may be determined by difference, as was done in problem 21. In this case, however, the total weight of original materials includes the added sugar. The computation, therefore, becomes: (6500 pounds of whole milk + 487.12 pounds of skim milk + 1304.85 pounds of sugar required) − 3070.24 pounds of condensed milk = 5221.73 *pounds of water to be evaporated.*

Proof: The desired ratio of msnf to milkfat in the finished product has been shown (1) to be 2.31. Therefore, the milk that is to be condensed should,

after standardization, contain this balance. No subsequent change resulting from the addition or removal of water or sugar will alter this ratio; it will merely vary the degree of concentration. So the accuracy of the standardization may be proved by computing the ratio existing in the standardized milk.

The fat content of the whole milk and of the added skim milk has been computed previously (2). The msnf content of the whole milk is 559 pounds (6500 pounds of whole milk x 8.6 percent msnf in whole milk = 559), and that of the skim milk is 43.8 pounds (487.12 pounds of skim milk x 9 percent msnf in skim milk = 43.8). The composition of the standardized and sugared milk before condensing is shown by the tabulation below:

	Pounds	Pounds of fat	Pounds of msnf
Original milk	6500.00	260.00	559.00
Added skim milk	487.12	0.97	43.80
Sugar	1304.85	.....	.....
Total weight	8291.97	260.97	602.80

Computing the msnf to milkfat ratio in the standardized milk, 602.8 ÷ 260.97 = 2.31 shows an existing ratio of 2.31 parts of msnf to each 1 part of milkfat, and proves the accuracy of the standardization.

Further checks may be made upon the basis of the solids constituents as related to the finished weight, as follows:

(1304.85 ÷ 3070.24) x 100 = 42.5 desired percent of sugar
(260.97 ÷ 3070.24) x 100 = 8.5 desired percent of milkfat
(602.80 ÷ 3070.24) x 100 = 19.6 desired percent of msnf

Algebraic Method of Solving Condensed Milk Standardization Problems

Condensed milk standardization problems can be solved easily by the use of simple algebraic expressions involving only one unknown, the weight of standardizing material needed. Consider problem 21 on page 11. The problem was to make plain condensed milk from 6500 pounds of whole milk testing 3.5 percent fat and 8.8 percent msnf. The resulting condensed milk product must contain 7.9 percent milkfat and 18.2 percent msnf. Available for standardizing are cream, 40 percent fat and 5.4 percent msnf, and skim milk containing 0.2 percent milkfat and 9 percent msnf. How much of which standardizing material is needed?

Solution: The first step is the same as in problem 21. Find the msnf to fat ratio in the milk (2.51) and in the desired product (2.30). This shows that additional fat is needed and cream will be used as the standardizing material. Let X = the weight of cream needed.

Write an equation for the weight of fat that will be present in the batch of standardized milk after the X pounds of cream have been added.

Total weight of fat = weight of whole milk x percent fat in whole milk
+ percentage of fat in cream x weight of cream
= 6500 x 0.035 + 0.40X

Write another equation for the total weight of msnf.

Total weight of msnf = weight of whole milk x percentage of msnf in whole milk + percentage of msnf in cream x weight of cream

$$= 6500 \times 0.088 + 0.054X$$

Since we want to select the weight of cream (X pounds) so that the total weight of msnf in the standardized batch divided by total weight of fat will be 2.30, we can divide the equation we have for the weight of msnf by the equation for the weight of fat, set this equal to the desired ratio of 2.30 and solve for X.

$$\frac{0.088 \times 6500 + 0.054X}{0.035 \times 6500 + 0.40\ X} = 2.30$$

See Appendix for method of solving this type of equation.

$$\frac{572 + 0.054X}{227.5 + 0.40X} = 2.30$$

$$572 + 0.054X = 523.25 + 0.92X$$

$$48.75 = 0.866X$$

$$X = 56.28 \text{ pounds of 40 percent cream}$$

The proof of this problem is the same as that given for problem 21.

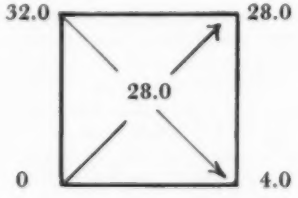
Standardizing Over-Condensed Batches of Condensed Milk by the Addition of Water

The Pearson's square can be used in standardizing over-condensed batches of condensed milk by the addition of water. It is necessary to know the weight of the over-condensed batch and the test of one of the constituents in it, as well as the desired test of that same constituent in the correctly finished product. The percentage of total solids is ordinarily used since it can be estimated simply by means of a hydrometer. Water, the standardizing material, is taken as having zero percent total milk solids.

Example 23

How much water should be added to an over-condensed batch of condensed whole milk weighing 4920 pounds and containing 32 percent total solids if the desired total solids content is 28 percent?

Solution:



For each 28 pounds of the material containing 32 percent total solids, we need 4 pounds of water (0.0 percent T.S.). Therefore, for 4920 pounds we need $\frac{4920}{28}$ x 4, or 703 pounds of water.

Proof:

4920 pounds x 0.32	=	1572
703 pounds x 0.0	=	0
5623		1572

$$\frac{1572 \times 100}{5623} = 28.0 \text{ percent total solids}$$

BUTTER

Moisture Control in Buttermaking

Butter must contain at least 80 percent fat. The other constituents are water, salt, and curd. Some butter is not salted.

After churning, a moisture test is run before the butter is removed from the churn and the amount of water, if any, needed for standardization is determined.

The calculations to show how much water is needed are complicated by the fact that there is no way to determine exactly how much butter there is in the churn. However, it is practical to make certain assumptions and the formula below gives satisfactory results.

Formula IV

Amount of water to add to unfinished butter in churn = Pounds of fat in churn x (DM-M) x 0.015

Where: DM = percentage of moisture desired in finished butter,
and M = percentage of moisture in unfinished butter (as determined by analysis)

Example 24

1940 pounds of 33 percent cream are churned and the amount of moisture in the butter is found by analysis to be 16 percent. The desired moisture in the finished butter is 17 percent. How much water must be added to the unfinished butter in the churn?

Solution: 1940 x 0.33 = 640.20 pounds of fat in churn

Pounds of water to add to churn = 640.20 x (17–16) x 0.015 = 9.6 lbs.

A very accurate method of calculation may be made as follows: 1940 pounds of 33 percent cream is to be made into butter. The moisture test on the unfinished butter is 16 percent. The desired composition of the butter is 80.2 percent fat, 2 percent salt, 0.8 percent curd, and 17 percent water.

How much water must be added for standardization?

1. 1940 pounds 33 percent cream to be churned.
2. 1940 x 0.33 = 640.20 pounds fat.

3. 640.20 x 0.0128 (percent of fat lost) = 8.20 pounds fat lost in buttermilk.
4. 640.20 − 8.20 = 632 pounds fat made into butter.
5. 632 ÷ 0.802 = 788 pounds of 80.2 percent butter to be made.
6. Desired (and expected) composition:

80.2 percent fat	x 788	=	732 pounds fat
2.0 percent salt	x 788	=	15.76 pounds salt
0.8 percent curd	x 788	=	6.3 pounds curd
17.0 percent water	x 788	=	133.96 pounds water
100.0 percent butter	x 788	=	788.00 pounds butter

7. Therefore, 788 x 0.02 = 15.76 pounds salt to add.
8. 16 percent water found by analysis in the unfinished butter.
9. Therefore, 100 percent − 16 percent = 84 percent fat, salt, and curd in the unfinished butter.
10. (632 pounds fat + 15.76 pounds salt + 6.3 pounds curd) ÷ 0.84 = 778.6 pounds unfinished butter.

Answer:

11. 788 pounds finished butter − 778.6 pounds unfinished butter = 9.4 pounds of water to be added for standardization

CHEESE

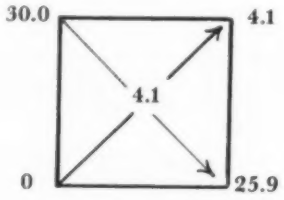
Creaming and Salting of Cottage Cheese

Creamed cottage cheese, by law, must contain at least 4.0 percent fat, by weight. To calculate the amount of cream needed to add to the uncreamed curd the Pearson's square method can be used.

Example 25

How many pounds of 30 percent cream are required to produce 259 pounds of creamed cottage cheese having 4.1 percent fat?

Solution:



259 ÷ 25.9 = 10
4.1 x 10 = 41 pounds of 30 percent cream
259 + 41 = 300 pounds of *unsalted* creamed cottage cheese

If approximately 0.8 percent salt is desired then: 0.008 x 300 = 2.4 pounds of salt needed. The percentage of salt in the product will be slightly less than 0.8 because the total weight of the product becomes 302.4 rather than 300 when the salt is added. The exact calculation can be done by Pearson's

square or by simple algebra. However, the approximation, as given here, is ordinarily close enough for the practical case.

The cheese was standardized to 4.1 percent fat to give a small margin of safety over the legal limit of 4 percent. The addition of the 2.4 pounds of salt will reduce the fat percentage to 4.07 percent.

The fat in the curd can be assumed as zero. The selection of the fat percentage in cream will depend upon the moisture content of the curd.

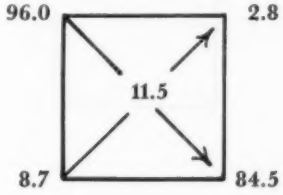
Fortifying Skim for Cottage Cheese

Non-fat dry milk or condensed skim are sometimes added to skim milk for cottage cheese to increase the total solids content. The amount of powder or condensed skim to add to the skim for cottage cheese can be calculated by the use of Pearson's square.

Example 26

How many pounds of non-fat dry milk containing 97 percent solids must be added to 200 pounds of skim milk containing 8.7 percent solids to raise the total solids content to 11.5 percent?

Solution:



200 ÷ 84.5 = 2.37

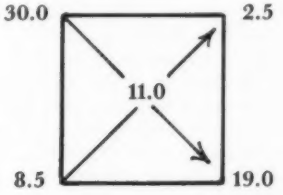
2.8 x 2.37 = 6.64 pounds of powder needed

In this case a certain amount of skim is fortified and the total amount of milk made into cheese is not considered important.

Example 27

How many pounds of skim milk containing 8.5 percent solids and how many pounds of condensed skim milk containing 30.0 percent solids are needed to obtain 3000 pounds of fortified skim containing 11 percent solids?

Solution:



3000 ÷ 21.5 = 139.53
2.5 x 139.53 = 348.83 pounds of condensed skim needed
19.0 x 139.53 = 2651.07 pounds of skim needed
348.83 + 2651.07 = 2999.90 pounds of fortified skim
In this case, an exact amount of fortified skim is prepared.

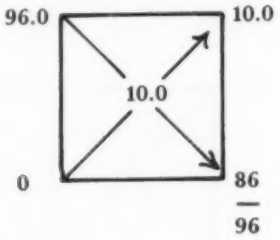
Making Cottage Cheese from Nonfat Dry Milk

Nonfat dry milk, or condensed skim, plus water can be used to prepare skim milk for cottage cheese making. The calculations can be made by the Pearson's square.

Example 28

How many pounds of nonfat dry milk testing 97 percent solids-not-fat and water must be mixed together to obtain 1200 pounds of skim milk testing 10 percent solids-not-fat?

Solution:



1200 ÷ 96 = 12.5
10 x 12.5 = 125.0 pounds of powder needed
86 x 12.5 = 1075.0 pounds of water needed
125 + 1075 = 1200.0 pounds of skim milk obtained

ICE CREAM

Ice cream mixes can be calculated by arithmetic or algebraic methods. With either method, the calculations are simplified by first finding the weights of ingredients needed to make 100 pounds of the desired mix. This is true because weights and percentages are the same on the basis of 100 pounds. The weights of ingredients needed for larger quantities of mix can then be determined by simple multiplication.

Since ice cream contains a higher percentage of fat than fluid milk, one of the mix ingredients must be high in fat. Cream, plastic cream, or butter is usually used for this purpose. Ice cream also contains more solids-not-fat than milk and a source high in these solids, such as condensed skim milk, condensed whole milk, or milk powder, is needed. Fluid milk is usually used as the basic dairy ingredient in ice cream mix. Calculation of the mix involves finding the required weights of these ingredients along with the necessary sugar and stabilizer.

The Arithmetic Method of Standardizing Ice Cream Mixes

This method involves the application of a few simple formulas for calculating the weight of the concentrated solids-not-fat ingredient and the weight of the high-fat ingredient. The formulas are modifications of those first presented by Cutter[2], whose method is commonly referred to as the "serum point" method. The word serum is used to refer to the fat-free portion of a dairy product. Serum solids (S.S.) are the milk-solids-not-fat in the dairy product. To simplify writing the formulas given below, the word "condensed" is used in place of "concentrated serum solids ingredient," the word "cream" is used in place of "concentrated fat ingredient," and the word "milk" is used in place of whole milk or skim milk, whichever is being used. The formulas are based on the assumption that normal skim milk contains 9 percent serum solids. This value is approximately correct and its use does not lead to any serious errors in the calculations.

The weight of condensed is calculated from the formula:

$$\text{Weight of condensed} = \frac{\text{pounds S.S. needed} - (0.09 \times \text{pounds serum in mix})}{\text{pounds S.S. in 1 pound condensed} - (0.09 \times \text{pounds serum in 1 pound condensed})}$$

The weight of cream is calculated from the next formula, which is applied to conditions as they exist after the proper weight of condensed has been added to the required amount of sugar, stabilizer and any other non-dairy ingredients.

$$\text{Weight of cream} = \frac{\text{pounds fat needed} - (\text{pounds milk and cream needed} \times \text{pounds fat in 1 pound milk})}{\text{pounds fat in 1 pound cream} - \text{pounds fat in 1 pound milk}}$$

Example 29

Calculate the weights of ingredients needed for 100 pounds of mix containing:

10 percent fat
10 percent serum solids
16 percent sugar
0.4 percent stabilizer

If the available ingredients are:

Plain condensed skim milk 30 percent S.S.
Cream 40 percent fat
Skim milk 9 percent S.S.
Sugar
Stabilizer

First calculate the pounds of serum in the mix

$= 100 - (\text{pounds fat} + \text{pounds sugar} + \text{pounds stabilizer})$
$= 100 - (10 + 16 + 0.4)$
$= 100 - 26.4$
$= 73.4$

[2]Cutter, T. D., "Proportioning Mixes Made Easy," *Ice Cream Trade Journal* 18:9, 45–50, 1922.

Use the first formula to calculate the weight of plain condensed skim:

$$\text{Weight of condensed} = \frac{10 - (0.09 \times 73.4)}{0.30 - (0.09 \times 1.00)}$$

(Note that the skim milk is assumed to contain no fat and hence the pounds of serum in 1 pound of it is 1).

$$= \frac{100 - 6.606}{0.3 - 0.09}$$

$$= \frac{3.494}{0.21}$$

= 16.64 pounds

The mix now contains:

16.64 lbs. plain condensed skim
16.0 lbs. sugar
0.4 lb. stabilizer

33.04 lbs.

The balance, or 100 – 33.04 = 66.96 pounds will be supplied by milk and cream.

$$\text{Weight of cream} = \frac{10 - (66.96 \times 0)}{0.4 - 0} = \frac{10}{0.4} = 25 \text{ lbs.}$$

Weight of skim milk = weight of milk + cream needed – weight of cream
= 66.96 – 25
= 41.96

It is wise to check the calculations by determining the weights of fat and S.S. supplied by each of the several ingredients.

Ingredient	Weight	Pounds fat	Pounds S.S.	Pounds sugar	Pounds stabilizer
Pl. cond. skim	16.64		4.99		
*Cream	25.00	10.0	1.35		
Skim milk	41.96		3.78		
Sugar	16.00			16.0	
Stabilizer	0.40				0.4
	100.00	10.0	10.12	16.0	0.4

*See miscellaneous section for method of calculating the S.S. content of cream.

Example 30

A mix made from sweet butter, nonfat dry milk and whole milk.
Make 100 pounds of mix containing:
14 percent fat
10 percent S.S.
15 percent sugar
0.3 percent stabilizer

from:

Sweet butter	82 percent fat
Nonfat dry milk	97 percent S.S.
Milk	4 percent fat

When applying the arithmetic method to mixes made from butter, a slight error is introduced because the serum portion of butter does not have the same composition as that of other dairy ingredients for ice cream. The error, however, is small and can be neglected.

Calculate the pounds of serum in the mix $= 100 - (14 + 15 + 0.3)$
$= 100 - 29.3$
$= 70.7$

The weight of serum in 1 pound of nonfat dry milk = 0.97 pound

$$\text{Weight of nonfat dry milk needed} = \frac{10 - (0.09 \times 70.7)}{0.97 - (0.09 \times 1.0)}$$

$$= \frac{10 - 6.363}{0.97 - 0.09}$$

$$= \frac{3.637}{0.88}$$

$$= 4.13 \text{ pounds}$$

The mix now consists of:

4.13	pounds nonfat dry milk
15.0	pounds sugar
0.3	pounds stabilizer
19.43	pounds

The remainder, 100 – 19.43 = 80.57 pounds will be supplied by a mixture of butter and milk.

$$\text{Weight of butter} = \frac{14 - (80.57 \times 0.04)}{0.82 - 0.04} = \frac{14 - 3.22}{0.78}$$

$$= \frac{10.78}{0.78} = 13.82 \text{ pounds}$$

Weight of milk = 80.57 – 13.82 = 66.75 pounds.

Ingredient	Weight	Pounds fat	Pounds S.S.	Pounds sugar	Pounds stabilizer
Dry skim	4.13		4.01		
Butter	13.82	11.32	0.22		
Milk	66.75	2.67	5.77		
Sugar	15.0			15.0	
Stabilizer	0.3				0.3
	100.0	13.99	10.00	15.0	0.3

Example 31

In this example the arithmetic method will be used to calculate an ice cream mix where all three dairy ingredients contribute both fat and S.S. to the mix and one of them also contains sugar.

Make 100 pounds of mix containing: 11 percent fat
11 percent S.S.
15 percent sugar
0.3 percent stabilizer

from:

Sweetened condensed whole milk	8 percent fat	20 percent S.S.	40 percent sugar
Cream	35 percent fat		
Whole milk	3.7 percent fat		
Sugar			
Stabilizer			

Solution:

Calculate the pounds of serum in the mix = 100 − (11 + 15 + 0.3)
= 100 − 26.3
= 72.7

Calculate the pounds of serum in 1 pound condensed
= 1 − (weight of fat + weight of sugar)
= 1 − (0.08 + 0.40)
= 1 − 0.48
= 0.52

Calculate the weight of condensed milk $= \dfrac{11 - (0.09 \times 72.7)}{0.20 - (0.09 \times 0.52)}$

$= \dfrac{11 - 6.543}{0.20 - 0.0468} = \dfrac{4.457}{0.1532} = 29.1$

The weight of fat and sugar the sweetened condensed milk will supply is:
29.1 x 0.08 = 2.33 pounds fat
29.1 x 0.40 = 11.64 pounds sugar

The weight of sugar still needed is:
15.00 − 11.64 = 3.36 pounds

The weight of fat still needed is:
11.00 − 2.33 = 8.67 pounds

If the additional sugar and the stabilizer are added to the condensed milk, the mix will consist of:

29.1 pounds condensed
3.36 pounds sugar
0.3 pounds stabilizer

32.76 pounds

The remainder (100 – 32.76 = 67.24 pounds) must be a mixture of milk and cream containing the 8.67 pounds fat still needed. Calculate the weight of cream needed:

$$= \frac{8.67 - (67.24 \times 0.037)}{0.35 - 0.037} = \frac{8.67 - 2.49}{0.313}$$

$$= \frac{6.18}{0.313} = 19.74$$

Weight of milk needed = 67.24 – 19.74 = 47.50 pounds

Summary

Ingredient	Weight	Pounds fat	Pounds S.S.	Pounds sugar	Pounds stabilizer
Sweetened condensed whole milk	29.10	2.43	5.82	11.64	
Cream	19.74	6.91	1.15		
Whole milk	47.50	1.76	4.12		
Sugar	3.36			3.36	
Stabilizer	0.3				0.3
	100.0	11.10	11.09	15.00	0.3

Algebraic Method of Standardizing Ice Cream Mixes

With this method it is not necessary to use formulas such as those used for the arithmetic method. Instead, three equations, involving the unknowns X, Y, and Z, where each of the symbols represents the weight of one of the dairy ingredients in the mix, are written for the particular mix desired and the ingredients from which it is to be made. The method is applicable to all types of mix problems and the mathematics involved in solving the three simultaneous equations is not difficult. Because of its general applicability, a little practice in becoming familiar with the method will be time well spent.

Mix made from cream and condensed skim milk

This example will be considered first because only two equations are required in the solution:

Example 32

Make 100 pounds of mix containing:
10 percent fat
10 percent S.S.
15 percent sugar
0.4 percent stabilizer

from:

Cream	40 percent fat	5.4 percent S.S.
Condensed skim milk	0.3 percent fat	30.0 percent S.S.
Sugar		
Stabilizer		
Water		

Water will be needed because no fluid milk is used.

Assign the symbol X to the weight of cream needed.

Assign the symbol Y to the weight of condensed skim needed.

Write an expression for the weight of fat contributed by cream and by the condensed skim milk and set the sum equal to the desired weight of fat in 100 pounds of finished mix.

(1) $0.40X + 0.003Y = 10$

Set the sum of the weights of serum solids contributed by the cream and condensed equal to the desired weight of serum solids.

(2) $0.054X + 0.30Y = 10$

(Consult the appendix for a discussion of the methods of solving simultaneous equations.)

Divide each equation by the coefficient of its X term to get:

$$(1)\ \frac{0.40X}{0.40} + \frac{0.003Y}{0.40} = \frac{10}{0.40}$$

$$(1)\ X + 0.0075Y = 25$$

$$(2)\ \frac{0.054X}{0.054} + \frac{0.304}{0.054} = \frac{10}{0.054}$$

$$(2)\ X + 5.56Y = 185.2$$

Change the sign of each term in the first equation and add it algebraically to the second equation.

$$(2)\quad X + 5.56Y = 185.2$$
$$(1)\quad -X - 0.0075Y = -25.0$$

$$5.5525Y = 160.2$$

$$Y = \frac{160.2}{5.5525} = 28.85 \text{ pounds}$$

Substitute this value into one of the equations and solve for X.

$$(1)\ X + 0.0075Y = 25$$
$$X + 0.0075\ (28.85) = 25$$
$$X + 0.22 = 25$$
$$X = 25 - 0.22$$
$$= 24.78 \text{ pounds}$$

The mix now contains:

28.85 pounds condensed skim milk
24.78 pounds cream
15.00 pounds sugar
0.40 pounds stabilizer

69.03 pounds

The balance, or 100 – 69.03 = 30.97 pounds should be water.

Ingredient	Weight	Pounds fat	Pounds S.S.	Pounds sugar	Pounds stabilizer
Cream	24.78	9.9	1.34		
Condensed skim	28.85	0.1	8.66		
Sugar	15.00			15.0	
Stabilizer	0.40				0.4
Water	30.97				
	100.00	10.0	10.00	15.0	0.4

The next example will illustrate, in detail, the steps involved in solving a mix problem involving 3 dairy ingredients.

Example 33

Make 100 pounds of mix containing:
10 percent fat
11 percent S.S.
13.5 percent sugar
0.5 percent stabilizer

from:

Cream containing	40 percent fat and 5.4 percent S.S.
Condensed skim milk	0.5 percent fat and 27.5 percent S.S.
Milk	4.0 percent fat and 8.6 percent S.S.
Sugar	
Stabilizer	

Assign the symbol X to the weight of cream needed.
Assign the symbol Y to the weight of condensed skim milk.
Assign the symbol Z to the weight of milk.

Write an equation involving the weights of fat contributed by the weights of the three dairy ingredients and set the sum equal to the desired weight of fat in the 100 pounds of finished mix. The X pounds of cream will supply 0.4X pounds of fat; the Y pounds of condensed skim will supply 0.005Y pounds of fat; and the Z pounds of milk will supply 0.04Z pounds of fat. Since 100 pounds of this mix should contain 10 pounds of fat, the first equation becomes:

(1) $0.4X + 0.005Y + 0.04Z = 10$

Next write an expression for the sum of the serum solids supplied by the dairy ingredients and set this equal to the desired weight of serum solids. This is the second of the three equations necessary for solving the problem.

(2) $0.054X + 0.275Y + 0.086Z = 11$

In the finished mix 13.5 + 0.5 = 14.0 pounds will be sugar and stabilizer. The rest of the mix, or 100 – 14 = 86 pounds will be made up of cream, condensed skim milk, and milk. The sum of the weights of these ingredients is, therefore, 86 pounds and this is the basis for the third equation.

(3) $X + Y + Z = 86$

The first step in solving these simultaneous equations is to solve the third one for Z in terms of X and Y and substitute this value of Z into equation (1) and (2). Equation (3) can be rewritten as:

$Z = 86 - X - Y$

Substituting this in (1) we obtain:

(1) $0.4X + 0.005Y + 0.04\ (86 - X - Y) = 10$

Multiplying, and combining like terms gives:

(1) $0.4X + 0.005\ Y + 3.44 - 0.04X - 0.04Y = 10$

(1) $0.36X - 0.035Y = 6.56$

Substituting the value of Z into the second equation gives:

(2) $0.054X + 0.275Y + 0.086\ (86 - X - Y) = 11$

(2) $0.054X + 0.275Y + 7.396 - 0.086X - 0.086Y = 11$

(2) $-0.032X + 0.189Y = 3.604$

The problem is now reduced to one of solving two simultaneous equations. This can be done most easily by dividing each equation by the coefficient of the X term.

Equation (1) becomes:

$$(1)\ \frac{0.36X}{0.36} - \frac{0.035Y}{0.36} = \frac{6.56}{0.36}$$

(1) $X - 0.097Y = 18.22$

Equation (2) becomes:

$$(2)\ \frac{-0.032X}{0.032} + \frac{0.189Y}{0.032} = \frac{3.604}{0.032}$$

(2) $-X + 5.906Y = 112.63$

Equations (1) and (2) are added algebraically to eliminate the X term.

(1) $X - 0.097Y = 18.22$

(2) $-X + 5.906Y = 112.63$

$$5.809Y = 130.85$$

Solve for Y:

$$Y = \frac{130.85}{5.809}$$

$Y = 22.53$ pounds

Substitute this value into equation (1) and solve for X.

$X - 0.097Y = 18.22$

$X - 0.097\ (22.53) = 18.22$

$X = 18.22 + 2.19$

$X = 20.41$ pounds

Substitute these values into equation (3) and solve for Z:

(3) $Z = 86 - X - Y$

$= 86 - 20.41 - 22.53 = 43.06$ pounds

SUMMARY

Ingredient	Pounds	Pounds fat	Pounds S.S.	Pounds sugar	Pounds stabilizer
X cream	20.41	8.16	1.10		
Y cond. skim milk	22.53	0.13	6.20		
Z milk	43.06	1.72	3.70		
sugar	13.50			13.5	
stabilizer	0.50				0.5
	100.00	10.01	11.00	13.5	0.5

Example 34

Make 100 pounds of mix containing:
12 percent fat
10 percent S.S.
15 percent sugar
0.3 percent stabilizer

From:

Cream	40 percent fat	5.4 percent S.S.	
Sweetened condensed whole milk	0.5 percent fat	27.5 percent S.S.	42 percent sugar
Milk	3.8 percent fat	8.6 percent S.S.	
Sugar			
Stabilizer			

The sugar and stabilizer in 100 pounds of mix will be $15 + 0.3 = 15.3$ pounds

The dairy ingredients should therefore weigh $100 - 15.3 = 84.7$ pounds

Let X = pounds cream
Y = pounds sweetened condensed whole milk
Z = pounds milk

Equation for the fat in the mix is:

(1) $0.4X + 0.005Y + 0.038Z = 12$

Equation for the S.S. is:

(2) $0.054X + 0.275Y + 0.086Z = 10$

In writing the equation for the total weight of the dairy ingredients, provision must be made for the fact that not all of the sweetened condensed milk is a dairy product. Since 42 percent of it is sugar, only 58 percent is a dairy ingredient. In equation (3), the term 0.58Y appears rather than Y.

(3) $X + 0.58Y + Z = 84.7$

(3) $Z = 84.7 - X - 0.58Y$

Substitute for Z in equation (1)

(1) $0.4X + 0.005Y + 0.038\ (84.7 - X - 0.58Y) = 12$

(1) $0.4X + 0.005Y + 3.22 - 0.038X - 0.022Y = 12$

(1) $0.362X - 0.017Y = 8.78$

Divide by the coefficient of X:

(1) $X - 0.047Y = 24.25$

Substitute for Z in equation (2):

(2) $0.054X + 0.275Y + 0.086\ (84.7 - X - 0.58Y) = 10$

(2) $0.054X + 0.275Y + 7.28 - 0.086X - 0.05Y = 10$

(2) $-0.032X + 0.225Y = 2.72$

Divide by the coefficient of X.

(2) $-X + 7.03Y = 85.00$

Add equations (1) and (2)

$$\begin{aligned} X - 0.047Y &= 24.25 \\ -X + 7.03Y &= 85.00 \\ \hline 6.98Y &= 109.25 \\ & \quad 109.35 \\ \hline Y &= 6.98 \\ &= 15.66 \text{ pounds} \end{aligned}$$

Substitute in equation (1) and solve for X.

$X - 0.047\ (15.66) = 24.25$

$X = 24.25 + 0.74$

$= 24.99$ pounds

Substitute in equation (3) and solve for Z:

$Z = 84.7 - 24.99 - 0.58\ (15.66)$

$= 84.7 - 24.99 - 9.08$

$Z = 50.63$

The weight of sweetened condensed milk (15.66 pounds) will supply

$0.42 \times 15.66 = 6.58$ pounds sugar

The balance, or $15.0 - 6.58 = 8.42$ pounds sugar will be in the form of dry sugar.

Summary

Ingredient	Weight	Pounds fat	Pounds S.S.	Pounds sugar	Pounds stabilizer
X cream	24.99	10.00	1.34		
Y sweetened condensed	15.66	0.08	4.31	6.58	
Z milk	50.63	1.92	4.35		
sugar	8.42			8.42	
stabilizer	0.30				0.30
	100.00	12.00	10.00	15.00	0.30

Overrun

The overrun in ice cream may be computed on either the weight or volume basis.

On the basis of weight, the formula is:

$$\text{Percent of overrun} = 100 \times \frac{(\text{Weight of given volume of mix} - \text{weight of same volume of ice cream})}{\text{Weight of same volume of ice cream}}$$

Example 35

What is the overrun in a sample of ice cream weighing 5 pounds per gallon if the mix from which it was made weighed 9 pounds per gallon?

Solution: Percent of overrun $= 100 \times \frac{(9 - 5)}{5}$

$= 100 \times \frac{4}{5}$

$= 80$ percent

On the basis of volumes, the formula is: percent of overrun =

$$100 \times \frac{(\text{volume of ice cream} - \text{volume of mix})}{\text{volume of mix}}$$

Example 36

If 5 gallons of ice cream mix yields 9.5 gallons of ice cream, what is the overrun in the ice cream?

Solution: percent of overrun $= 100 \times \frac{(9.5 - 5)}{5}$

$= 100 \times \frac{4.5}{5}$

$= 90$ percent

Weight of a Package of Ice Cream at Desired Overrun

It is often desirable to know what a package of ice cream will weigh when the desired overrun is reached. This can be calculated from a modified form of the first formula given for overrun. The formula is:

$$\text{Weight of given volume of ice cream} = 100 \times \frac{\text{weight of same volume of mix}}{100 + \text{percent of overrun}}$$

Example 37

What should a pint of ice cream weigh at 75 percent overrun if the mix being used weighs 9.2 pounds per gallon?

Solution: First find the weight of a gallon of the ice cream at 75 percent overrun and then find the weight per pint by dividing by 8, the number of pints in a gallon.

$$\text{Weight per gallon} = \frac{100 \times 9.2}{100 + 75}$$

$= \frac{920}{175}$

$= 5.26$ pounds

$= 84.1$ ounces

The weight of a pint should be $\frac{84.1}{8} = 10.5$ ounces

If this weight is being checked in the final package, the weight of the empty package should be added to the calculated weight of the ice cream.

Substituting Corn Sugar for Cane Sugar

At times it is desirable to replace part of the cane sugar in a mix with corn sugar in such a way that the sweetness of the mix will not be changed. To do this it is necessary to know the relative sweetness of the corn sugar compared to the cane sugar it is replacing.

Example 38

What weight of corn sugar should be used per 100 pounds of mix to replace 25 percent of the original 16 percent cane sugar? The relative sweetness of the corn sugar is 80 percent.

Solution: Weight of sugar to be replaced = 0.25 x 16 = 4 pounds

$$\text{Weight of corn sugar to use} = \frac{\text{Weight cane sugar to be replaced}}{\text{Relative sweetness, expressed as a fraction}}$$

$$= \frac{4}{0.8}$$

$$= 5.0 \text{ pounds}$$

The mix would then be made with (16 – 4) = 12 pounds of cane sugar and 5 pounds of corn sugar. The sweetness of the mix would not be changed in spite of the higher total sugar content.

MISCELLANEOUS

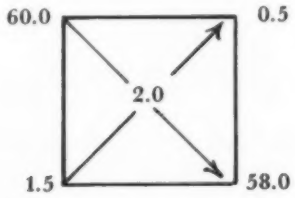
Other Applications of the Methods Presented for the Standardization of Milk and Cream

The Pearson's Square method can be applied to the standardization of other materials, provided only one constituent is involved.

Example 39

How many pounds of flake caustic of 60 percent purity must be added to 500 pounds of 1.5 percent caustic solution to bring the strength of the solution to 2.0 percent?

Solution:



For each 58.0 pounds of 1.5 percent solution we need 0.5 pound of 60 percent pure flake caustic. Therefore, for 500 pounds of the 1.5 percent solution we need $\frac{500}{58}$ or 8.62 times as much. The weight of flake caustic needed is therefore:

Answer: 8.62 x 0.5 = 4.31 pounds

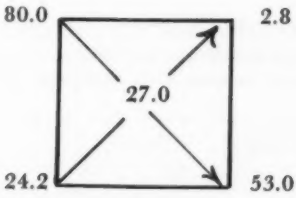
Proof: 500 pounds of 1.5 percent solution contains 500 x 0.015 = 7.50 pounds caustic.
4.31 pounds of 60 percent purity flake caustic contains 4.31 x 0.6 = 2.59
504.31 pounds of mixture contains 10.09 pounds.

$$\text{Percent caustic in mixture} = \frac{10.09}{504.31} \times 100 = 2.0 \text{ percent.}$$

Example 40

How many pounds of calcium chloride should be added to 2000 pounds of 24.2 percent brine to bring the strength to 27.0 percent? The crystalline $CaCl_2$ is 80 percent pure.

Solution:



For each 53 pounds of 24.2 percent brine, we need 2.8 pounds of 80 percent pure $CaCl_2$. Therefore, for 2000 pounds of 24.2 percent brine we need $\frac{2000}{53}$ or 37.74 times as much; or 37.74 x 2.8 = 105.7 pounds.

Proof: 2000 pounds 24.2 percent brine contains

$$\begin{array}{rl} 2000 \times 0.242 = & 484 \text{ pounds} \\ 105.7 \times 0.80 = & 84.5 \text{ pounds} \\ \hline & 568.5 \end{array}$$

$$\frac{568.5}{2105.6} \times 100 = 27.0 \text{ percent}$$

Hydrometers Used for Measuring the Specific Gravity of Condensed Milk Products

The most commonly used hydrometer for this purpose is the Baumé hydrometer. The Baumé scale starts at zero, which is the reading for water at the standard temperature of 60°F., and goes to values as high as 41 for very dense liquids. For liquids heavier than water, the Baumé value can be calculated from the specific gravity of the liquid at 60°F. by the following formula:

$$\text{Bé} = 145 - \frac{145}{\text{specific gravity}}$$

Calculation of Specific Gravity

The specific gravity of any dairy product, and hence its unit weight, can be calculated from its known composition and the specific gravities of the constituents. The following formula is used:

$$\text{Specific gravity at } 60°\text{F.} = \frac{100}{\frac{\text{percent Fat}}{0.93} + \frac{\text{percent SNF}}{1.608} + \frac{\text{percent Sucrose}}{1.589} + \frac{\text{percent Water}}{1.0}}$$

This formula is often useful in calculating the weight of a new dairy product.

Example 41

What is the weight per gallon at 60°F. of a product containing 22 percent fat, 16 percent SNF, and 15 percent sucrose?

Solution: Percent water in product = 100 − (percent F + percent SNF + percent sugar) = 47

$$\text{Specific gravity at } 60°\text{F.} = \frac{100}{\frac{22}{0.93} + \frac{16}{1.608} + \frac{15}{1.589} + 47}$$

$$= \frac{100}{23.66 + 9.95 + 9.44 + 47}$$

$$= 1.110$$

Since the density of water at 60°F. is 8.34 pounds per gallon, this dairy product will have a density of:

1.110 x 8.34 = 9.26 pounds/gallon

Weighted Average Test of Milk Received

If the weight and test of each of several lots of milk is known, the weighted average test, or test of the mixture obtained by blending all the lots, can be obtained by calculation. To do this, the total weight of fat in the several lots is divided by the total weight of the lots.

Example 42

Compute the weighted average test of the following lots of milk:

Producer	*Pounds of milk*	*Testing*
A	200	4.2
B	500	3.0
C	200	4.5
D	100	5.0
E	400	3.5

Solution:

	200 pounds x 0.042 =	8.4	pounds fat
	500 pounds x 0.030 =	15.0	pounds fat
	200 pounds x 0.045 =	9.0	pounds fat
	100 pounds x 0.050 =	5.0	pounds fat
	400 pounds x 0.035 =	14.0	pounds fat
Total	1400	51.4	

$$\text{Weighted average test} = \frac{\text{weight fat}}{\text{milk}} \times 100$$

$$= \frac{51.4}{1400} \times 100 = 3.67 \text{ percent}$$

SEPARATION

Calculating the Solids-not-fat Content of Cream Made from Milk of Known Composition

Milk consists of fat and skim milk. All of the solids-not-fat are in the skim milk portion. Cream is simply a mixture of fat and skim milk with considerably more fat than whole milk. The composition of the skim milk portion of the cream is the same as that of the skim milk portion of the milk from which it was made. The weight of solids-not-fat in one pound of skim milk can be found by dividing the percentage of solids-not-fat in the whole milk by the percentage of skim in the whole milk. Once this is known, the weight of solids-not-fat in one pound of cream can be found by multiplying the weight of skim milk in a pound of the cream. This can be converted to a percentage by multiplying by 100.

Example 43

What is the solids-not-fat content of 40 percent cream made from whole milk containing 4 percent fat and 8.6 percent solids-not-fat?

Solution: Percent skim in the milk = 100 − 4 = 96

$$\text{Pounds SNF in 1 lb. skim} = \frac{8.6}{96} = 0.0896$$

Pounds skim in 1 lb. cream = 1 − 0.4 = 0.6

Pounds SNF in 0.6 lb. skim = lb. SNF in 1 pound cream = 0.6 x 0.0896 = 0.0537

Percent SNF in cream = 5.37 percent

These relations can be combined into a single formula:

$$\text{Percent SNF in cream} = \frac{(100 - \text{percent fat in cream})}{(100 - \text{percent fat in milk})} \times \text{percent SNF in milk}$$

Applying the formula to the problem above we get:

$$\text{Percent SNF in cream} = \frac{(100 - 40)}{(100 - 4)} \times 8.6 = \frac{60}{96} \times 8.6 = 5.37 \text{ percent}$$

Estimating the Fat Content of Cream from the Fat Content of the Milk Being Separated

The method is based on the measurement of the relative flow rates of the skim milk and cream. This relative flow is usually determined on a volume basis. Because of this, the method gives only an approximate value for the fat content of the cream. To give a correct value, the relative flows would have to be determined on a weight basis because of the differences in the specific gravities of the milk and the cream. Since the method gives only an approximation, the measurement of relative flows can be made while the machine is being run on water.

Collect a quart of liquid from the skim spout. During the same interval that the quart is being collected, collect the discharge from the cream spout. Estimate the volume of cream in terms of the volume of skim. For example, the volume of cream may have been 1/10 the volume of skim. Take the denominator of this fraction, add 1 to it and multiply by the fat content of the milk being separated to obtain the fat content of the cream. If 4 percent milk was being separated in this example, the approximate fat content of the cream would be $4 \times (10 + 1) = 44$ percent.

Similar reasoning can be applied to determining beforehand what ratio of cream to skim should be used to get cream of a desired fat content.

Example 44

How should the separator be adjusted to get approximately 18 percent cream from 3.6 percent milk?

Solution: $\frac{18}{3.6} = 5$

$5 - 1 = 4$

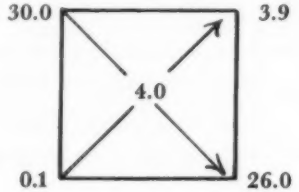
Therefore, the separator should be set so that the flow of cream is 1/4 that of skim.

Calculating the Yield of Cream from a Given Weight of Milk

Example 45

How much 30 percent cream can be obtained from 5000 pounds of 4 percent milk, if the skim milk contains 0.1 percent milkfat?

Solution: Set up the Pearson's square as indicated:



29.9

It shows that 29.9 pounds of 4 percent milk can be prepared by mixing 3.9 pounds of 30 percent cream with 26.0 pounds of 0.1 percent skim milk. This also means that 29.9 pounds of 4 percent milk, when separated, will yield 3.9 pounds of 30 percent cream and 26 pounds of 0.1 percent skim milk. Since we have 5000 pounds of 4 percent milk we will obtain $\frac{5000}{29.9}$ or 167.22 times as much cream as from 29.9 pounds of the milk. The yield of cream is therefore:

$$167.22 \times 3.9 = 652.16 \text{ pounds.}$$

The remainder, or 5000 – 652.16 pounds = 4347.84 pounds, is skim milk. This could also have been calculated in a manner similar to that used for the cream. The yield of skim from 5000 pounds of milk is 167.22 times that from 29.9 pounds or 167.22 x 26.0 = 4347.72 pounds.

Proof:

4347.72 pounds of 0.1 percent skim contains 4347.72 x 0.001 = 4.35 pounds fat

652.16 pounds of 30 percent cream contains 652.16 x 0.30 = 195.65 pounds fat

4999.88 pounds of mixture contain 200.00 pounds fat

$$\text{Percent fat in mixture} = \frac{200.00}{4999.88} \times 100 = 4.0 \text{ percent}$$

APPENDIX

Elementary Algebra Used in Some Dairy Calculations

1. Equations

Here is an equation: $A = B \times C$

The numerical value on either side of the equation is the same.

The value of an equation will not be altered if the same addition, subtraction, multiplication or division is applied to each side of the equation.

For example, divide both sides of the equation by C:

$$\frac{A}{C} = \frac{B \times C}{C}$$

The C's on the right side will cancel out leaving $\frac{A}{C} = B$

Or if both sides of the equation are divided by B:

$$\frac{A}{B} = \frac{B \times C}{B}$$

The B's on the right side of the equation cancel out leaving:

$$\frac{A}{B} = C$$

Now if we let A = weight of component
B = percent of component (written as a decimal)
C = weight of product
we have: A = B x C
weight of component = test x weight of product, or
weight of product x test = weight of component

Then rewriting $\frac{A}{C}$ = B or weight of component ÷ weight of product = percentage of component (written as a decimal).

Then rewriting $\frac{A}{B}$ = C or weight of component ÷ per cent of component = weight of product (written as a decimal).

II. Simultaneous Equations

The use of simultaneous equations makes the calculation of ice cream mix formulas relatively easy. A knowledge of elementary algebra is necessary. This information will be given and will then permit the reader to solve a problem as is done below:

Solve for X and Y. (In an ice cream mix problem these unknowns would represent dairy products such as milk and cream, or some other concentrated dairy product).

(1) $4X + 0.5Y + 2(4 - X - Y) = 95$
(2) $3X + 95.0Y + 5(4 - X - Y) = 110$
(1) $4X + 0.5Y + 8 - 2X - 2Y = 95$
(2) $3X + 95.0Y + 20 - 5X - 56 = 110$
(1) $2X - 1.5Y = 87$
(2) $-2X + 90.0Y = 90$

$88.5Y = 177$
Answer: $Y = 2$

To solve for X, substitute the value of Y into one of the equations:
$2X - 1.5Y = 87$
$2X - 1.5(2) = 87$
$2X - 3 = 87$
$2X = 90$
Answer: $X = 45$

To calculate the above simultaneous equation, the following knowledge of algebra is necessary:

A. Addition
Add the numbers when the signs are alike and use that sign.

+ 2	− 2
+ 4	− 4
+ 6	− 6

When signs are unlike add algebraically, ie., take the smaller no. from the larger no. and use the sign of the larger number.

− 2	+ 2
+ 4	− 4
+ 2	− 2

When there is no sign in front of a number it is understood as +. Therefore, + 2 could be written as simply 2.

B. Multiplication
With like signs the answer is +

+ 2	− 2
x + 4	x − 4
8	8

With unlike signs the answer is −

− 2	+ 2
x + 4	x − 4
− 8	− 8

C. Clearing of Parentheses ( )

If no number is in front of the parenthesis it is understood as +1. To remove parentheses, multiply everything inside the parentheses by the number just preceding it.

For example:

a. $(86 - x - 2y)$ is understood $+1\ (86 - x - 2y)$ or $86 - x - 2y$

b. $-2\ (86 - x - 3y)$ would equal

	86	$-x$	$3y$
	$x - 2$	$x - 2$	$x - 2y$
	-172	$+2x$	$+6y$

Solving simultaneous equations (with detailed explanations concerning each step).

Example 1

Solve for X and Y in these equations:

$X + 2Y = 10$

$3X - 4Y = 20$

We shall first solve for X as follows: Multiply the first equation by 2 so that the coefficients of the Y terms will be the same in both equations. This will eliminate the Y terms when the equations are added algebraically, provided the signs of the Y coefficients are opposite. If they are not opposite, they can be made opposite by multiplying each term in one of the equations by -1.

$2X + 4Y = 20$

The equations are now added algebraically and the Y terms will drop out. The value of X is found to be 8.

$2X + 4Y = 20$

$3X - 4Y = 20$

$5X = 40$

$$X = \frac{40}{5} = 8$$

Now solve for Y by substituting the value of X in one of the first simple equations. Then arrange the unknown term on one side of the equation and all the numerical terms on the other side of the equation.

$X + 2Y = 10$

$8 + 2Y = 10$

In this case it can be done by subtracting -8 from each side of the equation without changing the value of the equation.

$8 - 8 + 2Y = 10 - 8$

The 8's will drop out on the left-hand side of the equation

leaving: $2Y = 10 - 8$

and thus: $$Y = \frac{2}{2} = 1$$

Example 2

Solve for X and Y in these equations:

$2X + 3Y = 80$

$6X + 2Y = 100$

We shall first solve for X as follows: Multiply the first equation by 2, which is the coefficient of the Y term in the second equation. Multiply the second equation by 3, which is the coefficient of the Y term in the first equation.

(2) $2X + 3Y = 80$

$4X + 6Y = 160$

(3) $6X + 2Y = 100$

$18X + 6Y = 300$

The equations can now be added algebraically, but to have the Y terms drop out the signs of each Y term must be opposite. So in this problem each term in one of the equations must be multiplied by -1. This will change all the signs in one equation without changing the value of the equation.

$4X + 6Y = 160$

$18X + 6Y = 300$

$$-4X - 6Y = -160$$
$$18X + 6Y = 300$$

Adding algebraically we get: $14X = 140$

Therefore: $14X = 140$

$$X = \frac{140}{14} = 10$$

Now solve for Y as explained in example 1.

$$2X + 3Y = 80$$
$$2(10) + 3Y = 80$$
$$20 + 3Y = 80$$
$$3Y = 80 - 20$$
$$Y = \frac{60}{3} = 20$$

Example 3

Solve for X and Y in these equations:

$$4X + 0.5Y + 2(4-X-Y) = 95$$
$$3X + 95.0Y + 5(4-X-Y) = 110$$

First clear the parentheses as explained in paragraph C of this section.

$$4X + 0.5Y + 8 - 2X - 2Y = 95$$
$$3X + 95.0Y + 20 - 5X-5Y = 110$$

Next combine the X terms, the Y terms and the numerical terms respectively in each equation.

$$2X - 1.5Y = 87$$
$$22X + 90.0Y = 90$$

Adding algebraically we get: $88.5Y = 177$

Therefore, $Y = \frac{177}{88.5} = 2$

Next solve for X by substituting the value for Y in one of the two simplified equations.

$$2X - 1.5(2) = 87$$
$$2X - 3 = 87$$
$$2X = 90$$
$$X = \frac{90}{2} = 45$$

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